# **Nuclear matter properties in relativistic mean-field model with** *σ***-***ω* **coupling**

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**Abstract.** The  $\sigma$ - $\omega$  coupling is introduced phenomenologically in the linear  $\sigma$ - $\omega$  model to study the nuclear matter properties. It is shown that not only the effective nucleon mass  $M^*$  but also the effective  $\sigma$  meson mass  $m^*_{\sigma}$  and the effective  $\omega$  meson mass  $m^*_{\omega}$  are nucleon-density–dependent. When the model parameters are fitted to the nuclear saturation point, with the nuclear radius constant  $r_0 = 1.14$  fm and volume energy  $a_1 = 16.0 \text{ MeV}$ , as well as to the effective nucleon mass  $M^* = 0.85M$ , the model yields  $m^*_{\sigma} = 1.09m_{\sigma}$  and  $m^*_{\omega} = 0.90 m_{\omega}$  at the saturation point, and the nuclear incompressibility  $K_0 = 501 \text{ MeV}$ . The lowest value of  $K_0$  given by this model by adjusting the model parameters is around 227 MeV.

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## **1 Introduction**

As the starting point for the microscopic relativistic description of nuclear many-body system, within the framework of quantum hadrodynamics, the well-studied linear  $\sigma$ - $\omega$  model has been proved to be able to describe the saturation and other properties of nuclear matter [1]. However, this model yields the nuclear incompressibility  $K_0$  around 550 MeV which is unacceptably high, and also the effective nucleon mass  $M^*$  around 0.54 $M$  which seems uncomfortably low. In order to remedy this situation, many works have been done on the extension or generalization of this model [2]. Among others, the inclusion of nonlinear selfinteraction of  $\sigma$  mesons, introduced originally by Boguta and Bodmer [3], is shown to be successful in reducing the nuclear incompressibility  $K_0$  significantly. Along this line, the nonlinear self-interaction of  $\omega$  mesons has been included also in the relativistic mean-field theory [4]. On the other hand, the derivative scalar coupling, which contains higher powers of  $\sigma$  in couplings between nucleon and  $\sigma$ meson as well as a coupling between  $\sigma$  and  $\omega$  mesons, has been proposed and investigated extensively [5, 6]. Following these two lines, many types of coupling between nucleon,  $\sigma$ ,  $\omega$  and  $\rho$  mesons are included in the nuclear effective field theory by Furnstahl, Serot and Tang recently, in addition to the above-mentioned nonlinear self-interaction of  $\sigma$  and  $\omega$  mesons [7]. In this case, as the first step, it is worthwhile to investigate in more detail the effects of each type of coupling between nucleon,  $\sigma$ ,  $\omega$  and  $\rho$  mesons separately, before getting combined result by data fit to all of these couplings simultaneously.

The purpose of this paper is to investigate what is the effect on nuclear matter properties due to a  $\sigma$ - $\omega$  coupling, which is introduced phenomenologically and is equivalent to the  $\eta_2$  term of the Furnstahl et al.'s effective Lagrangian density, and especially whether this coupling is able to reduce the nuclear incompressibility  $K_0$  and in the same time to increase the effective nucleon mass  $M^*$  in relativistic mean-field theory. Section 2 gives the model and field equations. Section 3 presents the nuclear matter equation of state and related formulas for nuclear matter properties. The numerical calculation and results are given in sect. 4. Section 5 discusses the inclusion of  $\rho$  meson field, and Section 6 is the summary.

#### **2 Model and field equations**

Let us start with the following Lagrangian density:

$$
\mathcal{L} = \overline{\psi} [\gamma_{\mu} (i\partial^{\mu} - g_{\omega} \omega^{\mu}) - (M - g_{\sigma} \phi)] \psi \n+ \frac{1}{2} [(\partial_{\mu} - \eta g'_{\omega} \omega_{\mu}) \phi (\partial^{\mu} + \eta g'_{\omega} \omega^{\mu}) \phi - m_{\sigma}^{2} \phi^{2}] \n- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \omega_{\mu} \omega^{\mu} , \quad \eta = i \text{ and } 1, \tag{1}
$$

where  $F^{\mu\nu}=\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu},$   $\psi,$   $\phi$  and  $\omega$  are the nucleon,  $\sigma$  and  $\omega$  meson fields with masses M,  $m_{\sigma}$  and  $m_{\omega}$ , respectively, while  $g_{\sigma}$ ,  $g_{\omega}$  are the respective coupling constants, and  $g'_{\omega}$  is the  $\sigma$ - $\omega$  coupling constant.  $\eta = i$  gives a

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 $\sigma$ - $\omega$  coupling similar to that introduced by usual covariant derivatives, while  $\eta = 1$  gives a gauge-like virtual coupling similar to the imaginary coupling [8]. Mathematically, it is equivalent to add a term  $\eta^2 g_{\omega}^{\prime 2} \omega_{\mu} \omega^{\mu}$  to  $m_{\sigma}^2$ , which is just<br>the n<sub>o</sub> term introduced recently by Furnstabl *et al*, in their the  $\eta_2$  term introduced recently by Furnstahl *et al.* in their effective Lagrangian density  $\mathcal{L}_M$  (see eq. (52) of ref. [7]). According to this equivalence, the present work can be considered as a study of this  $\eta_2$  term in the Furnstahl et al.'s nuclear effective field theory. The present model reduces to the original Walecka model when the  $\sigma$ - $\omega$  coupling constant is zero,  $g'_{\omega} = 0$ . Physically, higher-order<br>terms of this  $\sigma_{\omega}$  coupling will give an effective nonlinear terms of this  $\sigma$ - $\omega$  coupling will give an effective nonlinear  $\sigma$  field self-interaction.

For static nuclear matter, the field equations derived from this Lagrangian density are reduced to the following equations in the mean-field approximation:

$$
(i\gamma^{\mu}\partial_{\mu} - g_{\omega}\gamma^{0}\omega_{0} - M^{*})\psi = 0, \qquad (2)
$$

$$
m_{\sigma}^{*2}\phi = g_{\sigma}\rho_s,\tag{3}
$$

$$
m_{\omega}^{*2}\omega_0 = g_{\omega}\rho_N,\tag{4}
$$

where the effective nucleon mass  $M^*$ , effective  $\sigma$  meson mass  $m^*_{\sigma}$  and effective  $\omega$  meson mass  $m^*_{\omega}$  are defined, respectively as spectively, as

$$
M^* = M - g_{\sigma}\phi,\tag{5}
$$

$$
m_{\sigma}^{*2} = m_{\sigma}^2 + \eta^2 g_{\omega}^{\prime 2} \omega_0^2, \tag{6}
$$

$$
m_{\omega}^{*2} = m_{\omega}^2 - \eta^2 g_{\omega}^{\prime 2} \phi^2, \tag{7}
$$

and  $\rho_s = \langle \overline{\psi} \psi \rangle$  is the scalar density,  $\rho_N = \langle \overline{\psi} \gamma^0 \psi \rangle$  the baryon density. For calculating these effective masses the baryon density. For calculating these effective masses, the above three equations can be rewritten as the following self-consistent equations:

$$
\xi = \frac{M^*}{M} = 1 - \frac{\alpha}{s} \frac{\rho_s}{\rho_0},\tag{8}
$$

$$
s = \xi_{\sigma}^{2} = \frac{m_{\sigma}^{*2}}{m_{\sigma}^{2}} = 1 + \frac{\alpha_{\sigma}}{v^{2}} \frac{\rho_{N}^{2}}{\rho_{0}^{2}},
$$
(9)

$$
v = \xi_{\omega}^{2} = \frac{m_{\omega}^{*2}}{m_{\omega}^{2}} = 1 - \frac{\alpha_{\omega}}{s^{2}} \frac{\rho_{s}^{2}}{\rho_{0}^{2}},
$$
 (10)

where  $\rho_0$  is the standard nucleon number density,  $\alpha$ ,  $\alpha_{\sigma}$ and  $\alpha_{\omega}$  are the dimensionless composite parameters defined, respectively, as

$$
\alpha = \frac{g_{\sigma}^2 \rho_0}{m_{\sigma}^2 M}, \quad \alpha_{\sigma} = \frac{\eta^2 g_{\omega}^{\prime 2} g_{\omega}^2 \rho_0^2}{m_{\omega}^4 m_{\sigma}^2}, \quad \alpha_{\omega} = \frac{\eta^2 g_{\omega}^{\prime 2} g_{\sigma}^2 \rho_0^2}{m_{\sigma}^4 m_{\omega}^2}.
$$
 (11)

It should be noted that  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  are positive for  $\eta = 1$ while negative for  $\eta = i$ . It is worthwhile to note that  $\alpha_{\sigma}$ and  $\alpha_{\omega}$  are proportional to the Furnstahl *et al.*'s  $\eta_2$ . For example, it can be written as

$$
\alpha_{\sigma} = -\frac{\eta_2}{2} \left( \frac{g_{\sigma} g_{\omega} \rho_0}{m_{\sigma} m_{\omega} M} \right)^2.
$$
 (12)

#### **3 Nuclear matter properties**

The nuclear matter equation of state derived from Lagrangian density (1) can be expressed in terms of the nuclear energy density  $\mathcal E$  as  $e = \mathcal E/\rho_N - M$ , and

$$
\mathcal{E} = \mathcal{E}_k + \mathcal{E}_\sigma + \mathcal{E}_\omega,\tag{13}
$$

$$
\mathcal{E}_k = \frac{M^4 \xi^4}{\pi^2} \sum_{i=p,n} F_1(k_i/\xi M), \tag{14}
$$

$$
\mathcal{E}_{\sigma} = \frac{1}{2}(1 - \xi)M\rho_s,\tag{15}
$$

$$
\mathcal{E}_{\omega} = \frac{1}{2}(1-\xi)yM\rho_s, \quad y = \frac{(s-1)(2v-1)}{s(1-v)},\tag{16}
$$

where  $k_p$  and  $k_n$  are the proton and neutron Fermi momenta, respectively, and the function  $F_m(x)$  is defined as [9]

$$
F_m(x) = \int_0^x dx \, x^{2m} \sqrt{1 + x^2}.
$$
 (17)

The baryon density  $\rho_N$  and scalar density  $\rho_s$  can be expressed as

$$
\rho_N = \frac{1}{3\pi^2} \sum_{i=p,n} k_i^3,
$$
\n(18)

$$
\rho_s = \frac{M^3 \xi^3}{\pi^2} \sum_{i=p,n} f_1(k_i/\xi M), \tag{19}
$$

where the function  $f_m(x)$  is defined as [9]

$$
f_m(x) = \int_0^x dx \frac{x^{2m}}{\sqrt{1+x^2}}.
$$
 (20)

Having the equation of state, the pressure  $p$  can be derived as

$$
p = -\mathcal{E} + \rho_N \frac{\partial \mathcal{E}}{\partial \rho_N} = \frac{1}{3} (\mathcal{E}_k - \mathcal{E}_\sigma - M\rho_s) + \mathcal{E}_\omega - \frac{2(1-s)}{s} \mathcal{E}_\sigma.
$$
\n(21)

Instead of Fermi momenta  $k_p$  and  $k_n$ , we will use the nucleon density  $\rho_N = \rho_n + \rho_p$  and relative neutron excess  $\delta = (\rho_n - \rho_p)/\rho_N$  as independent variables for describing the nuclear matter [10]. At the standard state  $\rho_N = \rho_0$ ,  $\delta = 0$ , the pressure should be zero,

$$
p\left(\rho_0, 0\right) = 0.\tag{22}
$$

In addition, the depth of the equation of state is related to the nuclear volume energy  $a_1$  as

$$
e(\rho_0, 0) = -a_1. \tag{23}
$$

The standard nucleon number density  $\rho_0$  defined by eq. (22) is related to nuclear radius constant  $r_0$  and Fermi momentum  $k_F$  of standard nuclear matter as  $\rho_0 =$  $3/4\pi r_0^3 = 2k_{\rm F}^3/3\pi^2$ .

The generalized nuclear incompressibility  $K(\rho_N, \delta)$ can be defined [11] and derived as

$$
K \equiv 9 \frac{\partial p}{\partial \rho_N} = \frac{3(p+\mathcal{E}) + 12\mathcal{E}_{\omega}}{\rho_N} - \left[3M\rho_s + \frac{12(3-2s)}{s}\mathcal{E}_{\sigma}\right]
$$

$$
\times \frac{1}{\rho_s} \frac{\partial \rho_s}{\partial \rho_N} + \frac{12(3-s)}{s} \frac{\mathcal{E}_{\sigma}}{\partial \rho_N} \frac{\partial s}{\partial \rho_N} - \frac{12(1-v)}{1-2v} \frac{\mathcal{E}_{\omega}}{v} \frac{\partial v}{\partial \rho_N}, \quad (24)
$$

$$
\rho_N \frac{\partial s}{\partial \rho_N} = -2(1-s) \Big[ 1 - \frac{2(1-v)}{(1-\xi)v} \rho_N \frac{\partial \xi}{\partial \rho_N} \Big],\tag{25}
$$

$$
\rho_N \frac{\partial v}{\partial \rho_N} = \frac{2(1-v)}{1-\xi} \rho_N \frac{\partial \xi}{\partial \rho_N},\tag{26}
$$

$$
\rho_N \frac{\partial \rho_s}{\partial \rho_N} = Q + 3(\rho_s - Q) \frac{\rho_N}{\xi} \frac{\partial \xi}{\partial \rho_N},\tag{27}
$$

$$
\rho_N \frac{\partial \xi}{\partial \rho_N} = -\frac{1}{s} \Big[ 2(1-\xi)(1-s) + \frac{\alpha Q}{\rho_0} \Big] \times \Big[ 1 - \frac{4(1-s)(1-v)}{sv} + \frac{3(1-\xi)(\rho_s - Q)}{\xi \rho_s} \Big]^{-1}, \tag{28}
$$

$$
Q = \frac{M^3 \xi^3}{3\pi^2} \sum_{i=p,n} \frac{k_i}{\xi M} f'_1(k_i/\xi M). \tag{29}
$$

The usual nuclear incompressibility  $K_0$  can be obtained from  $K(\rho_N, \delta)$  as

$$
K_0 = K(\rho_0, 0) = 9\left(\rho_N^2 \frac{\partial^2 e}{\partial \rho_N^2}\right)_0, \tag{30}
$$

the subscript 0 stands for the standard state  $\rho_N = \rho_0$  and  $\delta = 0$ . In addition, the following formula for symmetry energy J can be derived:

$$
J \equiv \frac{1}{2} \frac{\partial^2 e}{\partial \delta^2} \Big|_0 = \frac{1}{6} \frac{k_{\rm F}^2}{\sqrt{k_{\rm F}^2 + M^2 \xi_0^2}}.
$$
 (31)

This formula is the same as that given by the usual Walecka model [12, 13].

Around the standard state  $\rho_N = \rho_0, \delta = 0$ , the nuclear equation of state and thus the nuclear matter properties are specified essentially by the standard density  $\rho_0$ , volume energy  $a_1$ , symmetry energy J, incompressibility  $K_0$ , density symmetry L and symmetry incompressibility  $K_s$  [10], where  $L$  and  $K_s$  are, respectively,

$$
L \equiv \frac{3}{2} \left( \rho_N \frac{\partial^3 e}{\partial \rho_N \partial \delta^2} \right)_0 = \frac{3}{2\rho_0} \frac{\partial^2 p}{\partial \delta^2} \Big|_0, \tag{32}
$$

$$
K_s \equiv \frac{9}{2} \left( \rho_N^2 \frac{\partial^4 e}{\partial \rho_N^2 \partial \delta^2} \right)_0 = -6L + \frac{1}{2} \frac{\partial^2 K}{\partial \delta^2} \Big|_0. \tag{33}
$$



**Fig. 1.**  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  as function of  $\alpha$ . The point  $\alpha_{W} = 0.4908$ indicated by an arrow corresponds to the usual Walecka model where  $\alpha_{\sigma} = \alpha_{\omega} = 0$ .

#### **4 Numerical calculation and results**

For the description of nuclear matter, there are three independent parameters  $\alpha$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  in the present model. The saturation point of nuclear matter  $(\rho_0, a_1)$  defined by eqs. (22) and (23) can be used as input to determine  $\alpha_{\sigma}$ and  $\alpha_{\omega}$  as function of  $\alpha$ . The way is as follows. At the standard state  $(\rho_N = \rho_0, \delta = 0)$ , these two equations can be solved for  $s$  and  $y$ ,

$$
s = \frac{3(1 - \xi)M\rho_s}{3\rho_0(M - a_1) - 2\mathcal{E}_k - \xi M\rho_s},
$$
(34)

$$
y = \frac{2\rho_0(M - a_1) - 2\mathcal{E}_k - (1 - \xi)M\rho_s}{(1 - \xi)M\rho_s},
$$
(35)

and thus from eq. (16)

$$
v = \frac{sy + s - 1}{sy + 2s - 2}.\tag{36}
$$

For given  $\rho_0$  and  $a_1$ , s and v can be calculated by eqs. (34)-(36) for a chosen  $\xi$ . Then  $\alpha$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  can be calculated by eqs. (8)-(10) for a chosen  $\xi$ . In our calculation, the nuclear radius constant  $r_0 = 1.14$  fm which corresponds to the standard nucleon number density  $\rho_0 = 0.161 \,\text{fm}^{-3}$ , the nuclear volume energy  $a_1 = 16.0 \,\text{MeV}$ , nucleon mass  $M\,=\,938.9\,\textrm{MeV}$  and constant  $\hbar c\,=\,197.327053\,\textrm{MeV}\cdot \textrm{fm}$ are used. It is worthwhile to note that the value  $r_0 =$ <sup>1</sup>.14 fm is obtained from the data fit to nuclear charge radii [14] extracted from the elastic electron scattering data [15].

Figure 1 plots  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  as function of  $\alpha$ . At the point  $\alpha = \alpha_W = 0.4908$  (indicated by an arrow), we have  $\alpha_\sigma=\alpha_\omega=0,$  and the present model reduces to Walecka



**Fig. 2.** The dimensionless effective nucleon mass  $\xi$ ,  $\sigma$  meson mass  $\xi_{\sigma}$  and  $\omega$  meson mass  $\xi_{\omega}$  as function of  $\alpha$ . At the point  $\alpha_W = 0.4908$  indicated by an arrow,  $\xi = 0.5437$  and  $\xi_\sigma = \xi_\omega = 1$ .



**Fig. 3.**  $K_0$ , J, L, and  $K_s$  as function of  $\alpha$ . At the point  $\alpha_W =$ 0.4908 indicated by an arrow,  $K_0 = 553 \,\text{MeV}$ ,  $J = 20.2 \,\text{MeV}$ ,  $L = 70.6 \,\text{MeV}$ , and  $K_s = 88 \,\text{MeV}$ . At the lower limit  $\alpha_{\text{min}} =$ 0.1037,  $K_0 = 227 \,\text{MeV}$ .

model. For  $\alpha > \alpha_W$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  are negative, which corresponds to  $\eta = i$ . For  $\alpha < \alpha_W$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  are positive, which corresponds to  $\eta = 1$ . There is a lower limit  $\alpha_{\rm min} = 0.1037$ , below this point there is no physical solution, as the effective  $\omega$  meson mass becomes imaginary,  $\xi_{\omega}^2$  < 0. Figure 2 shows the dimensionless effective  $\sigma$  meson mass  $\xi$  and mass  $\xi$  and mass  $\xi$ , the dimensionless effective  $\sigma$  meson mass  $\xi_{\sigma}$  and the dimensionless effective  $\omega$  meson mass  $\xi_{\omega}$  as function of  $\alpha$ . At the point  $\alpha_W$ ,  $\xi = 0.5437$  and  $\xi_{\sigma} = \xi_{\omega} = 1$ . From  $\alpha_W$  to the right,  $\xi$  decreases as  $\alpha$  increases, and we have  $\xi_{\sigma}$  < 1 and  $\xi_{\omega}$  > 1. From  $\alpha_W$  to the left,  $\xi$  increases as  $\alpha$ decreases, and we have  $\xi_{\sigma} > 1$  and  $\xi_{\omega} < 1$ . Figure 3 gives  $K_0$ , J, L, and  $K_s$  as function of  $\alpha$ . At  $\alpha_W$ ,  $K_0 = 553 \text{ MeV}$ ,  $J = 20.2 \,\text{MeV}, L = 70.6 \,\text{MeV}, \text{ and } K_s = 88 \,\text{MeV}.$  As  $\alpha$ decreases,  $K_0$  decreases at first and then increases slowly, passing through a very flat plateau, and finally decreases rapidly to a lower limit 227 MeV. J and L as well as  $K_s$ decrease slowly as  $\alpha$  decreases. It is worthwhile to note that  $K_s$  is negative in the low  $\alpha$  side, in opposition to what is obtained in the usual  $\sigma$ - $\omega$  model. Experimentally,  $K_s$  obtained from the isoscalar giant monopole resonance energy is between  $-566 \pm 1350$  and  $34 \pm 159$  MeV [16]. On the other hand,  $K_0$  increases to very high values rapidly as  $\alpha$  increases. In this aspect, the case of  $\eta = i$  is not acceptable.

The dimensionless effective nucleon mass  $\xi$  at the saturation point can be used as the third input to fix the parameter  $\alpha$ . For example, if  $\xi = 0.85$  at the saturation point, we have  $\alpha = 0.1822$ . Correspondingly we have  $\alpha_{\sigma}$  = 0.1134,  $\alpha_{\omega}$  = 0.2938,  $\xi_{\sigma}$  = 1.085,  $\xi_{\omega}$  = 0.895,  $K_0$  = 501 MeV,  $J = 13.8 \text{ MeV}$ ,  $L = 30.1 \text{ MeV}$ , and  $K_s = -38$  MeV, where the values of  $\xi_\sigma$  and  $\xi_\omega$  are those at the saturation point. Instead of  $\xi$ , the nuclear incompressibility  $K_0$  could be also used to fix  $\alpha$ . If  $K_0 = 250 \,\text{MeV}$  is chosen, it yields  $\alpha = 0.1050$ ,  $\alpha_{\sigma} = 0.00783$ ,  $\alpha_{\omega} = 2.642$ ,  $\xi = 0.939, \xi_{\sigma} = 1.295, \xi_{\omega} = 0.328, \text{ and } K_s = -31.2 \text{ MeV},$ where the values of  $\xi$ ,  $\xi_{\sigma}$  and  $\xi_{\omega}$  are those at the saturation point.

#### **5 Inclusion of the** *ρ* **meson field**

The symmetry energy  $J$  can be increased by including the  $\rho$  meson field in the present model. In this case, the additional term in the Lagrangian density is

$$
\mathcal{L}_{\rho} = -\frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \mathbf{b}_{\mu} \cdot \mathbf{b}^{\mu} - g_{\rho} \overline{\psi} \gamma_{\mu} \boldsymbol{\tau} \cdot \mathbf{b}^{\mu} \psi, \quad (37)
$$

where  $\mathbf{B}^{\mu\nu} = \partial^{\mu} \mathbf{b}^{\nu} - \partial^{\nu} \mathbf{b}^{\mu}$ ,  $\mathbf{b}^{\mu}$  is the  $\rho$  meson field with mass  $m_{\rho}$  and coupling constant  $g_{\rho}$ , and  $\tau$  are the isospin matrices. The resulting nucleon field equation, in the mean-field approximation for static nuclear matter, has an additional term  $-g_{\rho}\tau_3\gamma^0b_0$ :

$$
(i\gamma^{\mu}\partial_{\mu} - g_{\omega}\gamma^{0}\omega_{0} - g_{\rho}\tau_{3}\gamma^{0}b_{0} - M^{*})\psi = 0.
$$
 (38)

The equation for the  $\rho$  meson field is

$$
b_0 = -\frac{g_\rho \rho_N \delta}{m_\rho^2},\tag{39}
$$

while eqs. (3) and (4) are the same. The additional terms to the nuclear energy density, pressure, generalized incompressibility, symmetry and density symmetry energies are, respectively,

$$
\mathcal{E}_{\rho} = \frac{1}{2} \alpha_{\rho} M \rho_N^2 \delta^2 / \rho_0, \qquad p_{\rho} = \mathcal{E}_{\rho}, \qquad K_{\rho} = \frac{18 \mathcal{E}_{\rho}}{\rho_N},
$$
  

$$
J_{\rho} = \frac{1}{2} \alpha_{\rho} M, \qquad L_{\rho} = 3J_{\rho}, \qquad (40)
$$

where

$$
\alpha_{\rho} = \frac{g_{\rho}^2 \rho_0}{m_{\rho}^2 M} \tag{41}
$$

is the  $\rho$  meson dimensionless composite parameter. It is easy to prove that there is no contribution to  $K_s$  from the  $\rho$  meson field. It is worthwhile to note that the additional term  $J_{\rho}$  for the symmetry energy is the same as that given by the usual linear  $\sigma-\omega-\rho$  model [12,13].

It can be seen that, as  $\mathcal{E}_{\rho} = 0$  at the standard state where  $\delta = 0$ , the inclusion of  $\rho$  meson produces no change in nuclear energy density  $\mathcal{E}$ , pressure  $p$  and generalized incompressibility  $K$  at the standard state. Therefore, the parameters  $\alpha$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  have no change, and thus the dimensionless effective masses  $\xi$ ,  $\xi_{\sigma}$  and  $\xi_{\omega}$  are the same even if the  $\rho$  meson field is included. The parameter  $\alpha_{\rho}$ can be fixed by using the symmetry energy  $J$  as the fourth input, if parameters  $\alpha$ ,  $\alpha_{\sigma}$  and  $\alpha_{\omega}$  have been fixed.

## **6 Summary**

In conclusion, the  $\sigma-\omega$  coupling is introduced phenomenologically in the linear  $\sigma$ - $\omega$  model to study the nuclear matter properties. It is shown that, in comparison with the usual Walecka model, not only the effective nucleon mass  $M^*$  but also the effective  $\sigma$  meson mass  $m^*$  and<br>the effective  $\omega$  meson mass  $m^*$  are nucleon-densitythe effective  $\omega$  meson mass  $m^*_{\omega}$  are nucleon-density–<br>dependent. When the model parameters are fitted to dependent. When the model parameters are fitted to the nuclear radius constant  $r_0 = 1.14$  fm and volume energy  $a_1 = 16.0 \,\text{MeV}$  as well as to the effective nucleon mass  $M^* = 0.85M$ , the model yields  $m^*_{\sigma} = 1.09m_{\sigma}$  and  $m^* = 0.90m_{\sigma}$  at the same nuclear saturation point  $m^*_{\omega} = 0.90 m_{\omega}$  at the same nuclear saturation point,<br>and the nuclear incompressibility  $K_0 = 501 \text{ MeV}$ . This and the nuclear incompressibility  $K_0 = 501 \,\text{MeV}$ . This incompressibility seems too high. On the other hand, if the model parameters are fitted to  $K_0 = 250 \,\text{MeV}$ , in addition to  $r_0 = 1.14$  fm and  $a_1 = 16.0$  MeV, it yields  $M^* = 0.938M, m^*_{\sigma} = 1.295m_{\sigma}$  and  $m^*_{\omega} = 0.328m_{\omega}$  at

the nuclear saturation point. This effective  $\omega$  meson mass seems too low. In addition, the lower limit of incompressibility  $K_0 = 227 \,\text{MeV}$  is not low enough. Therefore, our conclusion is: Even if this  $\sigma-\omega$  coupling is able to reduce the nuclear incompressibility and also to increase the effective nucleon mass simultaneously, there is not yet enough degree of freedom to adjust the parameters in order to give a more reasonable result.

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